4/4/2016 الاثنين م. شوالب محاضرة [7]

State Space Approaches :-

1. Representation 2. Analysis 3. Design

For Linear Systems

$\dot{x} = Ax + Bu \rightarrow f(x,u)$ X: state vector

$y = CX + \cancel{D}u$ u: control input ; y: output

A, B, C, D System Matrices

In Discrete form

$X((K+1)T) = A_d X(KT) + B_d U(KT)$

$y(KT) = C_d X(KT) + D_d V(KT)$

$A_d \neq A \; ; \; C_d \neq C \; , B_d \neq B \; ; \; D_d \neq D \Leftarrow$

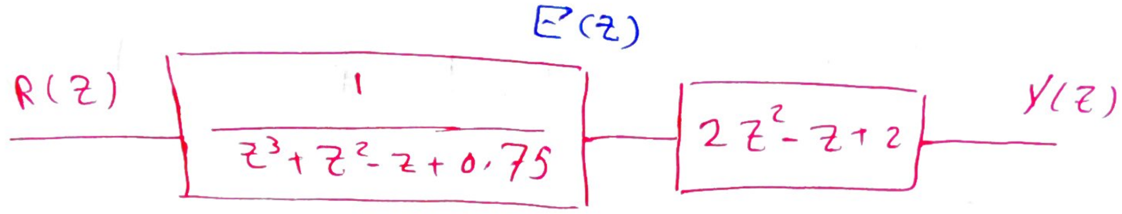
Representation of state variable Model:-

1. Controller Canonical form.
2. Observer Canonical form.
3. Parallel/Diagonal form.
4. Cascaded form.

Ex: Given $\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75}$

Required

1. Control canonical form.
2. Observer Canonical form.



1st Block

$$\frac{E(z)}{R(z)} = \frac{1}{z^3 + z^2 - z + 0.75}$$

$$\Rightarrow z^3 E(z) + z^2 E(z) - zE(z) + 0.75\, E(z) = R(z)$$

inverse $\Downarrow$

$$e(k+3) + e(k+2) - e(k+1) + 0.75\, e(k) = r(k)$$

Let $e(k) = x_1$; $e(k+1) = x_2$; $e(k+2) = x_3$

$\therefore$ $e(k+1) = x_1(k+1)$; $e(k+2) = x_2(k+1)$; $e(k+3) = x_3(k+1)$

$$e(k+3) = r(k) - e(k+2) + e(k+1) - 0.75\, e(k)$$

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = x_3(k)$$

$$x_3(k+1) = r(k) - x_3(k) + x_2(k) - 0.75\, x_1(k)$$

$$X(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.75 & 1 & -1 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k)$$

2nd Block

$$\frac{Y(z)}{E(z)} = 2z^2 - z + 2$$

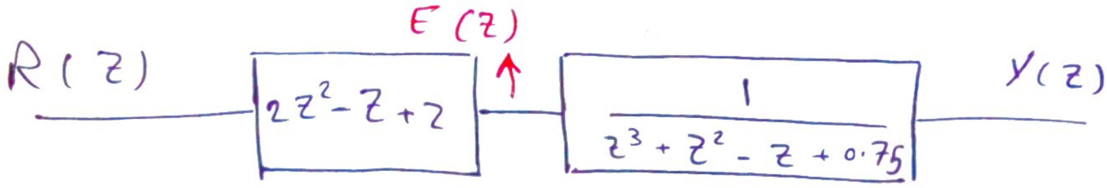
inverse $Y(z) = 2z^2 E(z) - zE(z) + 2E(z)$

$$y(k) = 2e(k+2) - e(k+1) + 2e(k)$$

Assumption : $x_1(k) = e(k)$ | $y = 2x_3(k) - x_2(k)$
$x_2(k) = e(k+1)$ | $+ 2x_1(k)$
$x_3(k) = e(k+2)$

$$y(k) = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} X$$

* Observer canonical form



1st block

$$\frac{E(z)}{R(z)} = 2z^2 - z + 2$$

$$\Rightarrow E(z) = 2z^2 R(z) - z R(z) + 2R(z)$$

inverse $\Downarrow$

$$e(k) = 2r(k+2) - r(k+1) + 2r(k)$$

2nd Block

$$\frac{Y(z)}{E(z)} = \frac{1}{z^3 + z^2 - z + 0.75}$$

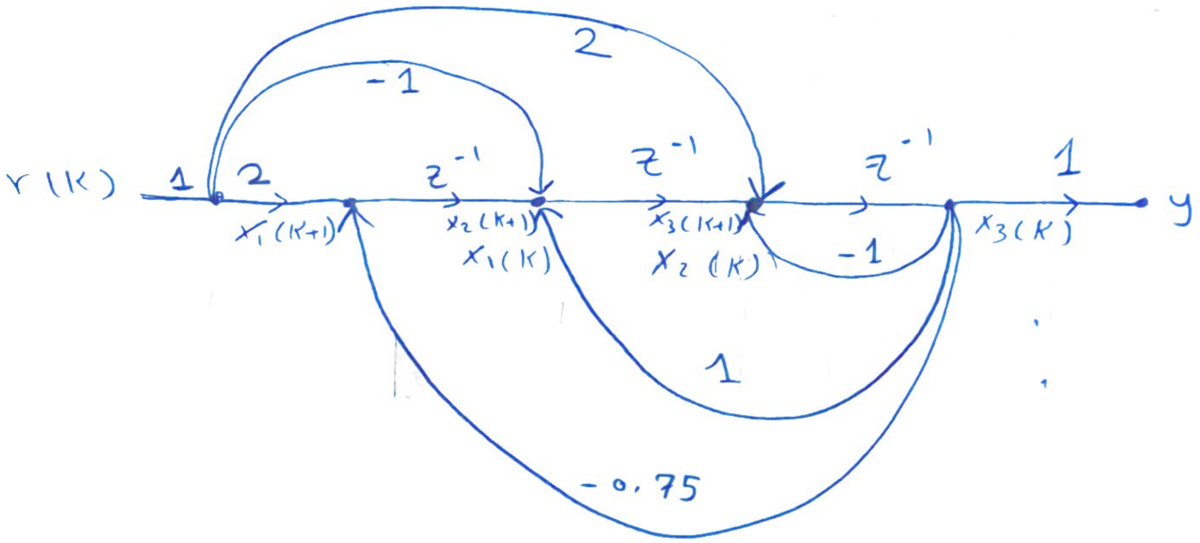
$$z^3 Y(z) + z^2 Y(z) - z Y(z) + 0.75\, Y(z) = E(z)$$

inverse $\Downarrow$

$$\underbrace{y(k+3)}_{x_3(k+1)} + y(k+2) - y(k+1) + 0.75\, y(k) = e(k)$$

$$\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75} * \frac{z^{-3}}{z^{-3}}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 + z^{-1} - z^{-2} + 0.75 z^{-3}} = \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - [-z^{-1} + z^{-2} - 0.75 z^{-3}]}$$



$$x_1(K+1) = -0.75\, x_3(K) + 2\, r(K)$$
$$x_2(K+1) = x_1(K) - r(K) + x_3(K)$$
$$x_3(K+1) = x_2(K) + 2\, r(K) - x_3(K)$$

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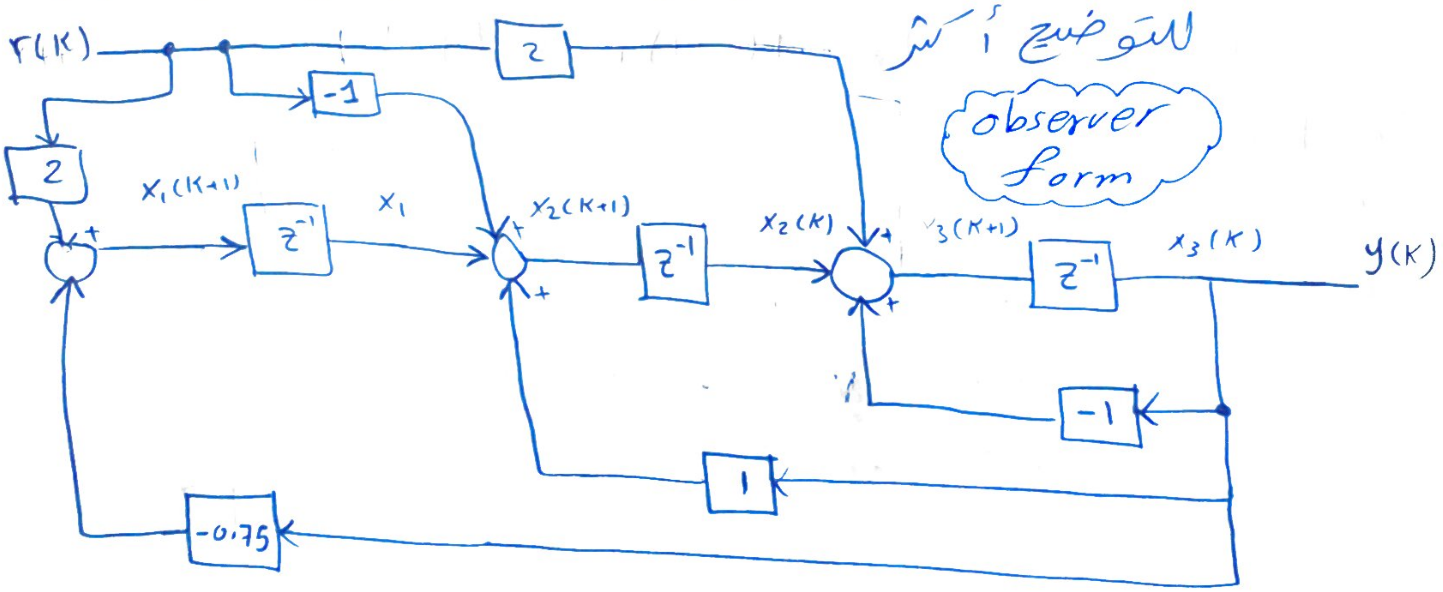
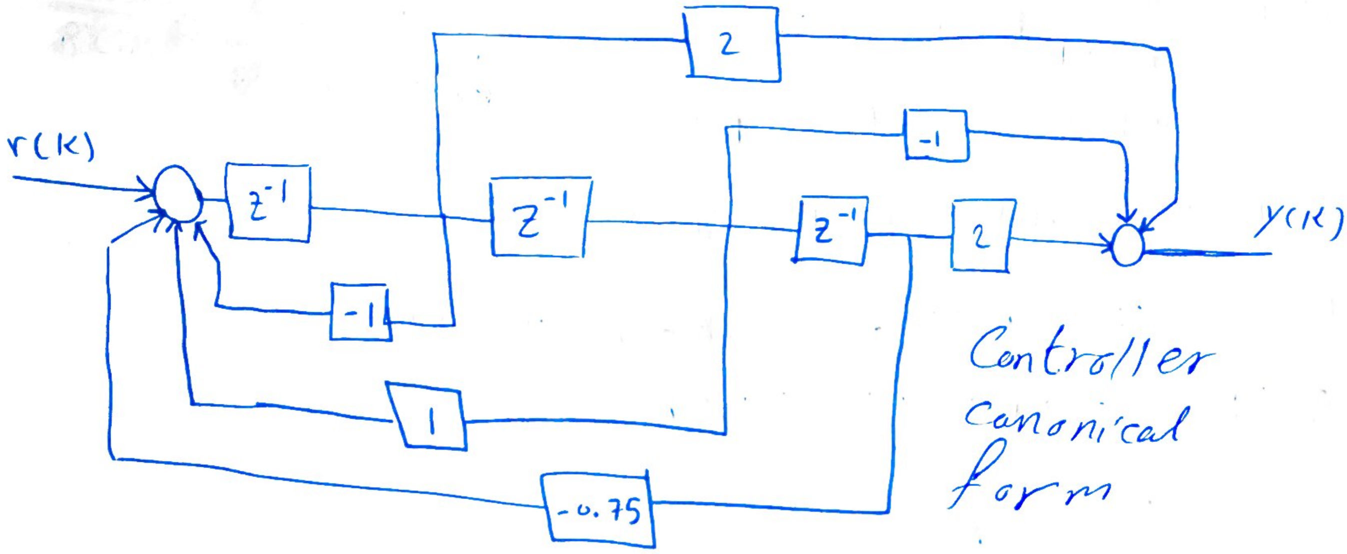
observer form



$$X(K+1) = \begin{bmatrix} 0 & 0 & -0.75 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} X(K) + \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} r(K)$$

$$Y(K) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X(K)$$

$$\frac{Y(z)}{R(z)} = \frac{\sum P_i}{1 - [\text{Loops}]}$$



Controller canonical form

* Given T.F.

$$\frac{Y(z)}{R(z)} = \frac{z+0.5}{(z-1)(z-0.1)(z+0.4)}$$

- Required: Representation in diagonal form

$$T.F. = \frac{A}{z-1} + \frac{B}{z-0.1} + \frac{C}{z+0.4}$$

$$= \frac{1.19}{z-1} + \frac{-1.333}{z-0.1} + \frac{0.143}{z+0.4}$$

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$$Y(z) = \frac{1.19}{z-1}R(z) + \frac{-1.333}{z-0.1}R(z) + \frac{0.143}{z+0.4}R(z)$$

$$Y_1 = \frac{1.19}{z-1}R(z) \Rightarrow z\,y_1(z) - Y_1(z) = 1.19\,R(z)$$

inverse ⇓

$$y_1(K+1) - y_1(K) = 1.19\,r(K)$$

Let $y_1(K) = x_1(K)$

$$x_1(K+1) = y(K+1)$$

$$x_1(K+1) = x_1(K) + 1.19\,r(K)$$

$$Y_2(z) = \frac{-1.33}{z-0.1} R(z)$$

$$z Y_2(z) - 0.1\, Y_2(z) = 1.33\, R(z)$$

$$\Downarrow$$

$$y_2(k+1) - 0.1\, y_2(k) = 1.33\, r(k)$$

Let $x_2(k) = y_2(k) \longrightarrow x_2(k+1) = y_2(k+1)$

$$x_2(k+1) = 0.1\, x_2(k) - 1.33\, r(k)$$

$$Y_3(z) = \frac{0.143}{z+0.4} R(z)$$

$$z Y_3(z) + 0.4\, Y_3(z) = 0.143\, R(z)$$

$$\Downarrow$$

$$y_3(k+1) + 0.4_3(k) = 0.143\, r(k)$$

Let $x_3 = y_3$

$$x_3(k+1) = y_3(k+1)$$

$$x_3(k+1) = -0.4\, x_3(k) + 0.143\, r(k)$$

$$X(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.4 \end{bmatrix} + \begin{bmatrix} 1.19 \\ -1.33 \\ 0.143 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} X(k)$$